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P.A. Gavrilov

SEISMIC ANALYSIS OF CRANES REGARDING GEOMETRIC AND PHYSICAL NONLINEARITIES

П.А. Гаврилов

УЧЕТ ФИЗИЧЕСКОЙ И ГЕОМЕТРИЧЕСКОЙ НЕЛИНЕЙНОСТЕЙ ПРИ РАСЧЕТЕ СЕЙСМОСТОЙКОСТИ КРАНОВЫХ МЕТАЛЛИЧЕСКИХ КОНСТРУКЦИЙ

The paper studies the problem of factoring geometric and physical nonlinearities in seismic analysis of cranes. An experimental study of factoring nonlinear characteristics of dynamics model was conducted. A series of mathematical models were developed to describe different ways of linearization of the experimental model. Comparison of results shows that physical nonlinearity of wire ropes is not required as factor in seismic analysis of cranes. Therefore, contact between wheel and rail and one-sided stiffness of wire ropes are required to be taken into account. Any liberalization of the dynamical model entails deviation of stresses, displacements and accelerations as results of the analysis.

SEISMIC ANALYSIS; CRANE; EXPERIMENTAL STUDY; NONLINEAR DYNAMICS MODEL; MECHANICS.

В работе рассматривается вопрос учета нелинейных характеристик динамических моделей при расчете сейсмостойкости грузоподъемных кранов общего назначения. Проведено экспериментальное исследование влияния геометрической и физической нелинейностей на параметры колебаний динамической системы. Составлен ряд математических моделей, описывающих колебательный процесс нелинейной динамической системы. Предложен метод аппроксимации нелинейных ступенчатых функций с разрывами в производных. Рассмотрены геометрически нелинейные математические модели, допускающие разрыв односторонних связей механической системы, а так же физически нелинейные системы. Проведен сравнительный анализ влияния различных нелинейностей на параметры колебательного процесса. Результаты исследования показывают, что линеаризация нелинейной механической системы приводит к значительным отклонениям количественных параметров колебаний и искажению их общего характера. При этом предлагаемая математическая модель показала удовлетворительную погрешность расчета нелинейных колебаний.

СЕЙСМОСТОЙКОСТЬ; КРАН ГРУЗОПОДЪЕМНЫЙ; ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ; НЕЛИНЕЙНАЯ ДИНАМИЧЕСКАЯ МОДЕЛЬ; МЕХАНИКА.

Introduction

Currently, calculating the seismic resistance of metal cargo crane structures that do not make up the equipment of nuclear power plants is regulated by [1] and [2]. Both of these documents establish possible methods for analyzing the seismic resistance of a structure: the eigenmode expansion (in accordance with Biot–Benioff's response spectrum theory of seismic resistance [3, 4]) and the direct integration of a system of differential equations. The response spectrum theory of seismic resistance allows to determine additional seismic loads on a structure using as input data the so-called response spectra of the foundation of a crane (a structure) and the magnitude of the design basis earthquake (DBE). The methodology of calculations by the response spectrum theory of seismic resistance was described in [5]. The input data for direct dynamical analysis of a structure are either recorded or synthesized accelerograms, velocigrams, or seismograms of earthquakes and the DBE magnitude.

In engineering practice, strength, stiffness and stability analysis of metal structures of cargo cranes is typically performed using the finite element method (FEM). Therefore, as computing power grows and calculation costs are reduced, direct dynamical methods are becoming increasingly used. Solving differential equations by numerical methods allows to more accurately determine the dynamical coefficients of displacements, stresses and strains, and as a result, to reduce the metal intensity and the construction costs of the crane.

However, the aforementioned regulations do not provide either specific guidance on choosing dynamical models for the seismic analysis of cranes, or recommendations on factoring in unilateral contacts (such as the rail-wheel contact, or hoist rope) and the nonlinear nature of the deformation of the suspended elements. This gap in the explanations can be attributed to the fact that the response spectrum theory of seismic resistance has been developed for building structures, where such nonlinearities occur much less frequently in metal cranes, and are therefore not taken into account in the calculations. However, these factors can play a very significant role for hoisting machines. The problem of taking into account the geometric and physical nonlinearities in seismic calculations of cargo cranes is being actively discussed in scientific circles [6–8]. The mathematical models and methods offered for solving these problems allow for potential bounces of the trolley wheels and the crane on the rail, unilateral contact of the ropes, physical nonlinearities of the structural materials, etc. The urgency of the research in this area is due to the need to improve the safety of hoisting machines operating in seismically hazar-dous zones.

This study is dedicated to investigating the influence of physical and geometrical nonlinearities in a dynamical system with regard to calculating the seismic resistance of cargo cranes. The ultimate goal of the study is in developing recommendations for taking into account the nonlinear properties of metal structures of general-purpose cargo cranes subjected to seismic analysis. The study consisted of the experimental part, mathematical simulation and the analysis of results.

Experimental setup

The experimental model simulates the construction of an overhead crane, sensing vertical oscillations and schematized as a geometrically and physically nonlinear dynamical three-mass system. A schematic for the experimental setup is shown in fig. 3. The main elements of the model are conventionally referred to as bridge *3*, trolley *4* and weight *6*. The bridge is secured in hinged supports *1* and *2*, with the movable support *2* allowing the bridge to move in the direction of its longitudinal axis. Strain gauges *7* are fixed to the bridge at the distance of a quarter of the span from the supports. The trolley is located at the center of the bridge span. The trolley can bounce on the bridge. If the trolley separates from the bridge, sensor *8* is triggered, registering the bounce event. Thus, the experiment simulates the contact between the trolley wheels and the rail. Weights 10 and accelerometer *9* measuring the vertical accelerations of the pendant emerging during oscillations are secured to the trolley. Guide ropes *12*, stretched along the vertical axis and passing through special holes in the trolley, are used to eliminate the swinging of the trolley from the vertical plane. A pendant, referred to as weight in the experiment, is fixed to the trolley by a flexible elastic suspension element. The pendant, as well as the trolley, is equipped with an accelerometer for recording accelerations and with additional calibrated weights. Before the start of the experiment the whole system was pulled downward and secured using hook *11* mounted to the lower suspension element. The measurements started at the moment when this constraint was removed (i.e., the thread connecting the weight to the foundation was cut).

The general characteristics of the experimental model were the following: the bridge span was 1,520 mm; the bridge weight was 0,503 kg; the mass of the suspended trolley with the accelerometer and additional weights was 1,9 kg; the mass of the suspended pendant with the accelerometer and additional weights was 2,63 kg.

In order to determine the actual stress-strain curve for the stretched flexible suspension element, a series of measurements was carried out for the elongation of the sample under various loads. Fig. 2 shows a plot for the stiffness of the flexible suspension element versus absolute elongation. The interpolating curve is described by the following expression with a conditional operator:

$$y(x) = \begin{cases} a + b \cdot e^{-c \cdot x} \text{ if } x \geq d; \\ C3 \text{ if } 0 \leq x < d; \\ 0 \text{ otherwise}, \end{cases} \qquad (1)$$

where a = 0,34 N/mm; b = 2,5 N/mm; c = 0,02; d = 82 mm; $C3$ = 0,87 N/mm.

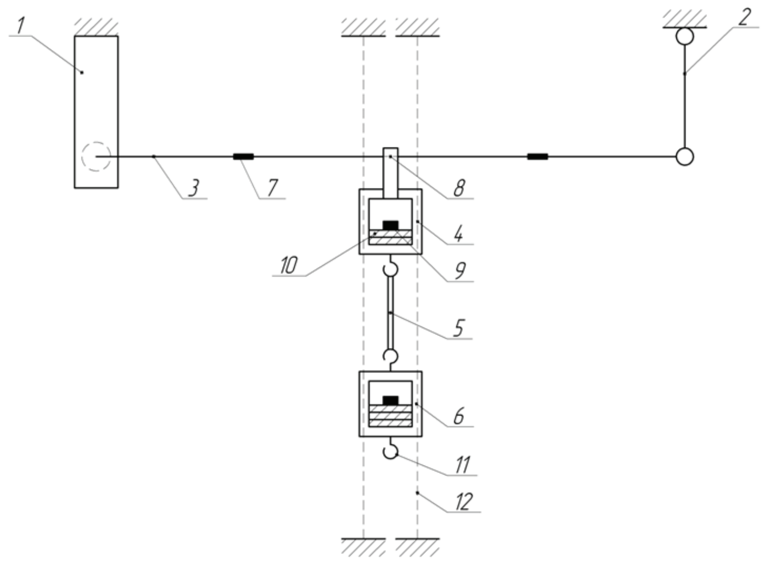

Fig. 1. Schematic of the experimental setup: fixed support *1*; movable support *2*; bridge *3*; suspended trolley *4*; flexible suspension element *5*; suspended ‘weight’ pendant *6*; strain gauge *7*; bounce sensor *8*; accelerometer *9*; additional weight *10*; hook *11*; guide ropes *12*

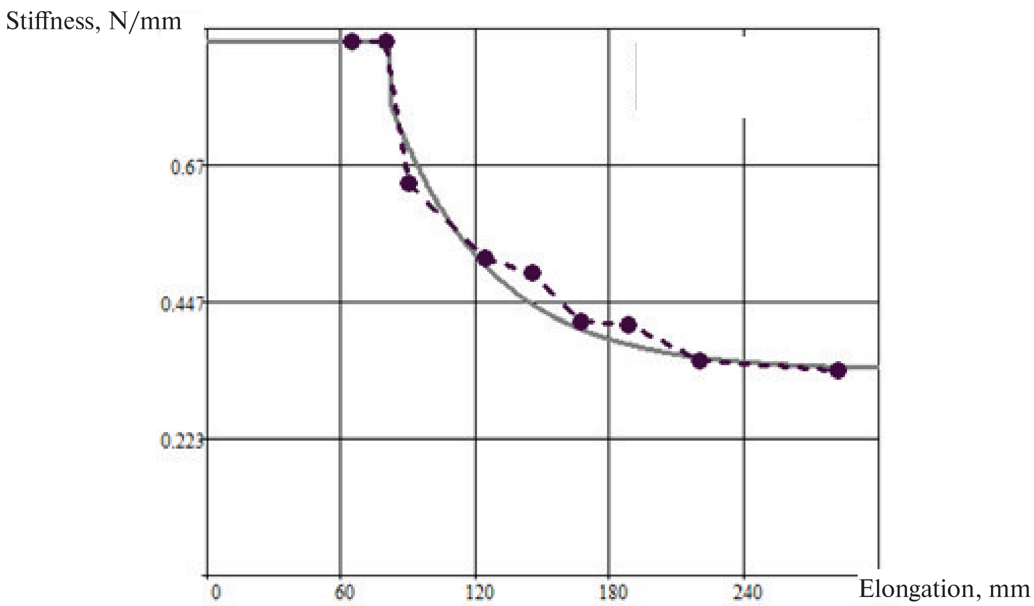

Fig. 2. Stiffness of the flexible suspension element versus elongation (——— interpolation, – – – experiment)

The results of the experiment are the recordings of the strain gauge, the bounce sensor and the accelerometers.

The experiment conducted demonstrated the effect of the geometric (weight and trolley bounce) and the physical (change in the stiffness of the flexible suspension element) nonlinearities.

Mathematical modeling

In accordance with the input experimental data, a number of mathematical models describing the vibrations of a dynamical three-mass system were constructed in MathCAD. Four models were designed: *1*—a linear dynamical model; *2*—a physically nonlinear dynamical model taking into account the actual nature of the tension of the flexible suspension element; *3*—a geometrically nonlinear dynamical model taking into account the possibility that the weight may bounce on the flexible suspension element and the trolley may bounce on the bridge; *4*—a physically and geometrically nonlinear dynamical model. The schematic for the dynamical three-mass system is shown in fig. 3.

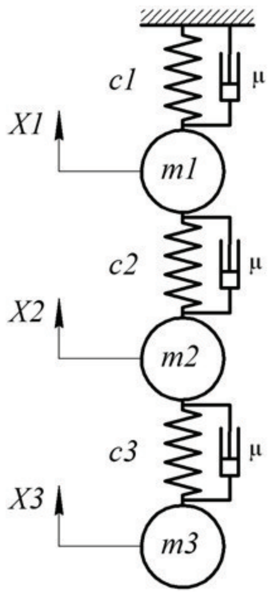

Fig. 3. Dynamical system schematic

The mathematical models are based on the Lagrange equation of the second kind, which has the form:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{R\}, \qquad (2)$$

where $[M]$ is the mass matrix of the structure, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix; $\{R\}$ is the known time-dependent vector of the external load; $\{u\}$ is the unknown time-dependent vector of mass displacements in the model.

The systems of differential equations were solved numerically in MathCAD 14 via the Radau solver which uses the eponymous algorithm. The process of energy dissipation during fluctuations is taken into account through equivalent viscous friction for all mathematical models. The coefficient of viscous friction was determined from the equality condition of the decay times for the oscillations of the experimental and mathematical models. For all of the mathematical models described below the coefficient of equivalent viscous friction was $\mu = 5$ kg /s.

The linear dynamical model is the simplest of all the ones presented. The system of differential equations describes the oscillations of the dynamical three-mass system.

$$\begin{cases} \ddot{X}1(t)\cdot m1 + c1\cdot X1(t) - c2\cdot(X2(t) - X1(t)) + \\ + \mu\cdot\dot{X}1(t) - \mu\cdot(\dot{X}2(t) - \dot{X}1(t))) = g\cdot m1; \\ \ddot{X}2(t)\cdot m2 + c2\,(X2(t) - X1(t)) - \\ -C3`\cdot(X3(t) - X2(t)) + \mu\cdot(\dot{X}2(t) - \dot{X}1(t)) - \\ - \mu\cdot(\dot{X}3(t) - \dot{X}2(t)) = g\cdot m2; \\ \ddot{X}3(t)\cdot m3 + C3`\cdot(X3(t) - X2(t)) + \\ + \mu\cdot(\dot{X}3(t) - \dot{X}2(t)) = g\cdot m3. \end{cases} \qquad (3)$$

Here $c1$ is the bridge stiffness, N/mm; $c2$ is the stiffness of the ‘trolley-bridge’ contact pair, N/mm; $c3$ is the equivalent stiffness of the flexible suspension element, N/mm; $X1(t)$, $X2(t)$ and $X3(t)$ are the generalized coordinates of the bridge, the trolley and the weight, respectively, mm; $m1$ is the reduced mass of the bridge, kg; $m2$ and $m3$ are the masses of the trolley and the weight, including the accelerometers, kg; g is acceleration of free fall, mm/s^2; μ is the coefficient of viscous friction, kg/s.

The stiffness of the flexible suspension element $c3$(t) is in this case taken to be constant. The greatest oscillation time is in the linear non-zero segment of the curve shown in fig. 2; however, the stiffness of the flexible suspension element for the initial time is calculated from Eq. (1). The equivalent stiffness used during the physical linearization of the model is taken as the average of the above values:

$$C3` = \frac{y(X2(0) - X3(0)) + C3}{2} = 0{,}68,\mathrm{N/mm}.$$

System (3) was solved with the following initial conditions:

$$\begin{cases} \dot{X}1(0) = 0,\mathrm{mm/s}; \\ \dot{X}2(0) = 0,\mathrm{mm/s}; \\ \dot{X}3(0) = 0,\mathrm{mm/s}; \\ X1(0) = -69{,}3\,\mathrm{mm}; \\ X2(0) = -69{,}3\,\mathrm{mm}; \\ X3(0) = -188{,}2\,\mathrm{mm}. \end{cases} \qquad (4)$$

In this study the geometric linearity was represented by the so-called buckling constraints. This means that the $c2$ and $c3$ stiffnesses only experienced uniaxial tension. Otherwise, the stiffnesses vanish and do not create reaction forces affecting the oscillation process. At the same time, given the specifics of the experimental setup, the dissipative elements retain their functions for any values of the relative displacements of the masses with respect to each other.

The variable stiffness is represented in the mathematical model as a continuous differentiable function:

$$c(x) = \frac{C}{\pi}\operatorname{arctg}\left(x \cdot k\right) + \frac{C}{2}, \tag{5}$$

where C is the maximum constant stiffness, N/mm; k is the coefficient determining the slope of the function.

The system of differential equations describing a geometrically linear dynamical system has the form:

$$\begin{cases} \ddot{X}1(t) \cdot m1 + c1 \cdot X1(t) - c2(t) \cdot (X2(t) - X1(t)) + \\ + \mu \cdot \dot{X}1(t) - \mu \cdot (\dot{X}2(t) - \dot{X}1(t))) = g \cdot m1; \\ \ddot{X}2(t) \cdot m2 + c2(t) \cdot (X2(t) - X1(t)) - \\ - c3(t) \cdot (X3(t) - X2(t)) + \mu \cdot (\dot{X}2(t) - \dot{X}1(t)) - \\ - \mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m2; \\ \ddot{X}3(t) \cdot m3 + c3(t) \cdot (X3(t) - X2(t)) + \\ + \mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m3; \\ c2(t) = \dfrac{C2}{\pi} \cdot \operatorname{arctg}((X1(t) - X2(t)) \cdot k) + \dfrac{C2}{2} \\ c3(t) = \dfrac{C3`}{\pi} \cdot \operatorname{arctg}((X2(t) - X3(t)) \cdot k) + \dfrac{C3`}{2}, \end{cases} \tag{6}$$

where $k = 1000$ is the dimensionless slope coefficient of the stiffness function.

System (6) was solved with the following initial conditions:

$$\begin{cases} \dot{X}1(0) = 0,\ \text{mm/s}; \\ \dot{X}2(0) = 0, \text{mm/s}; \\ \dot{X}3(0) = 0, \text{mm/s}; \\ X1(0) = -69{,}3, \text{mm}; \\ X2(0) = -69{,}3, \text{mm}; \\ X3(0) = -188{,}2, \text{mm}; \\ c2(0) = 10000, \text{N/mm}; \\ c3(0) = 0{,}68, \text{N/mm}. \end{cases} \tag{7}$$

The relative stiffness of the flexible suspension element under geometric linearization of the model is simulated according to Eq. (1), the difference being that the argument of the function is the absolute value of the argument:

$$yd(x) = \begin{vmatrix} a + b \cdot e^{-c \cdot |x|} \text{ if } |x| \geq d; \\ C3 \text{ otherwise}. \end{vmatrix} \tag{8}$$

The system of differential equations describing a physically nonlinear system has the form:

$$\begin{cases} \ddot{X}1(t) \cdot m1 + c1 \cdot X1(t) - c2 \cdot (X2(t) - X1(t)) + \\ + \mu \cdot \dot{X}1(t) - \mu \cdot (\dot{X}2(t) - \dot{X}1(t))) = g \cdot m1; \\ \ddot{X}2(t) \cdot m2 + c2 \cdot (X2(t) - X1(t)) - \\ - c3(t) \cdot (X3(t) - X2(t)) + \mu \cdot (\dot{X}2(t) - \dot{X}1(t)); \\ -\mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m2; \\ \ddot{X}3(t) \cdot m3 + c3(t) \cdot (X3(t) - X2(t)) + \\ + \mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m3; \\ c3(t) = \begin{vmatrix} a + b \cdot e^{-c \cdot (X1(t) - X2(t))} \text{ if } |X1(t) - X2(t)| \geq d; \\ C3 \text{ otherwise}. \end{vmatrix} \end{cases} \tag{9}$$

System (13) can be solved with the following initial conditions:

$$\begin{cases} \dot{X}1(0) = 0,\ \text{mm/s}; \\ \dot{X}2(0) = 0,\ \text{mm/s}; \\ \dot{X}3(0) = 0,\ \text{mm/s}; \\ X1(0) = -69{,}3,\ \text{mm}; \\ X2(0) = -69{,}3,\ \text{mm}; \\ X3(0) = -211{,}6,\ \text{mm}; \\ c3(0) = yd(X2(0) - X3(0)),\ \text{N/mm}. \end{cases} \tag{10}$$

The geometrically and physically nonlinear mathematical model contains all of these types of nonlinearities, and is described by a system of differential equations:

$$\begin{cases} \ddot{X}1(t) \cdot m1 + c1 \cdot X1(t) - c2(t) \cdot (X2(t) - X1(t)) + \\ + \mu \cdot \dot{X}1(t) - \mu \cdot (\dot{X}2(t) - \dot{X}1(t))) = g \cdot m1; \\ \ddot{X}2(t) \cdot m2 + c2(t) \cdot (X2(t) - X1(t)) - \\ - c3(t) \cdot (X3(t) - X2(t)) + \mu \cdot (\dot{X}2(t) - \dot{X}1(t)); \\ - \mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m2; \\ \ddot{X}3(t) \cdot m3 + c3(t) \cdot (X3(t) - X2(t)) + \\ + \mu \cdot (\dot{X}3(t) - \dot{X}2(t)) = g \cdot m3; \\ c2(t) = \dfrac{C2}{\pi} \cdot \operatorname{arctg}((X1(t) - X2(t)) \cdot k) + \dfrac{C2}{2}; \\ c3(t) = \begin{vmatrix} a + b \cdot e^{-c \cdot (X1(t) - X2(t))} \text{ if } (X1(t) - X2(t)) \geq d; \\ C3 \text{ if } 0 < (X1(t) - X2(t)) < d; \\ 0 \text{ otherwise}. \end{vmatrix} \end{cases} \tag{11}$$

System (11) can be solved with the following initial conditions:

$$\begin{cases} \dot{X}1(0) = 0, \text{ mm/s}; \\ \dot{X}2(0) = 0, \text{ mm/s}; \\ \dot{X}3(0) = 0, \text{ mm/s}; \\ X1(0) = -69{,}3, \text{ mm}; \\ X2(0) = -69{,}3, \text{ mm}; \\ X3(0) = -211{,}6, \text{ mm}; \\ c2(0) = 10000, \text{ N/mm}; \\ c3(0) = y(X2(0) - X3(0)), \text{ N/mm}. \end{cases} \quad (12)$$

Analysis of the results

As previously stated, the following parameters of the setup's oscillations were recorded during the experiment:

1) stresses on the surface of the rod in the points where the strain gauges were attached;
2) the vertical accelerations of the trolley;
3) the vertical accelerations of the weight;
4) the separation of the trolley from the bridge.

We should note that in this case, the stresses registered by the strain gauges are directly proportional to the displacements of the central point of the bridge span. Figs. 4–6 show comparative plots for the displacements of the central point of the bridge, and the accelerations of the trolley and the weight during oscillations.

In view of the requirements of engineering analysis, we compared the experimental results with the analytical calculations by the following parameters:

- the amplitude of the first wave of bridge displacement oscillations A;
- the duration of the bounce;
- the amplitude of the trolley acceleration oscillations excluding the first half-wave;
- the amplitude of weight acceleration oscillations excluding the first half-wave.

The results of the comparison of different mathematical models with the experimental data are listed in Table.

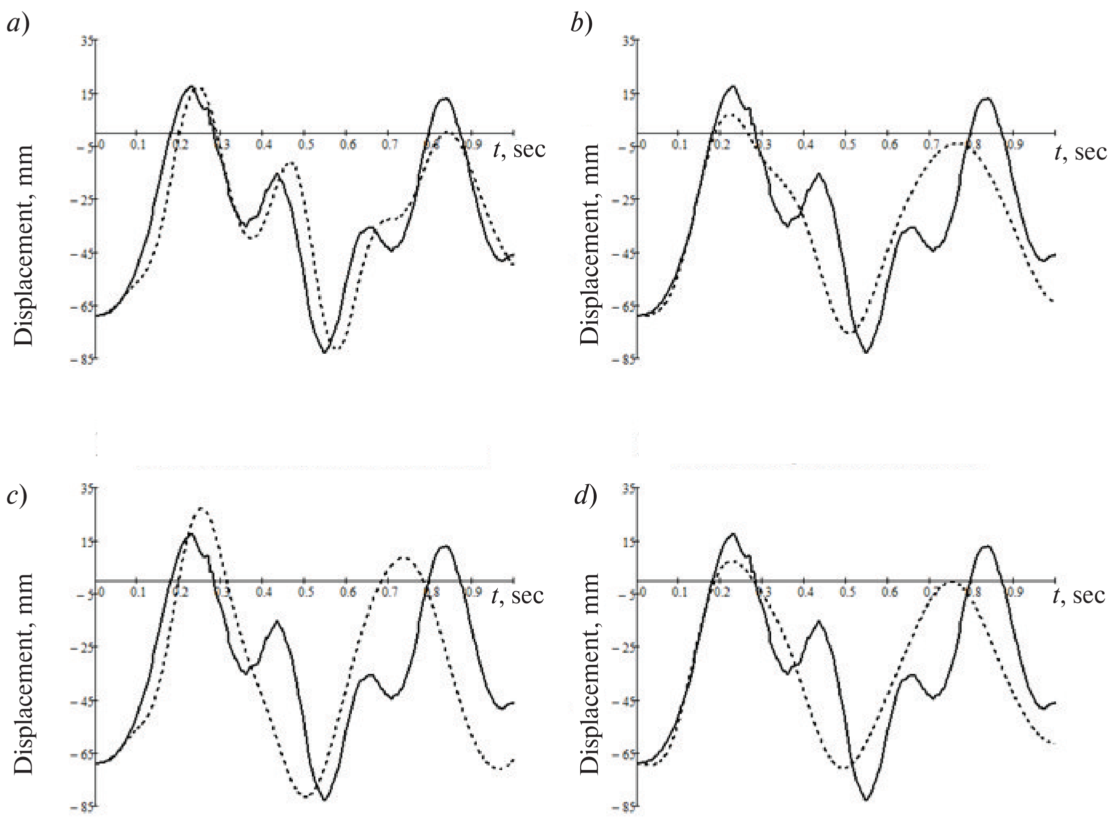

Fig. 4. Bridge displacements versus time: ——— experiment; ············ model (*a* – physicall and geometricall nonlinear; *b* – geometricall nonlinear; *c* – physicall nonlinear; *d* – linear)

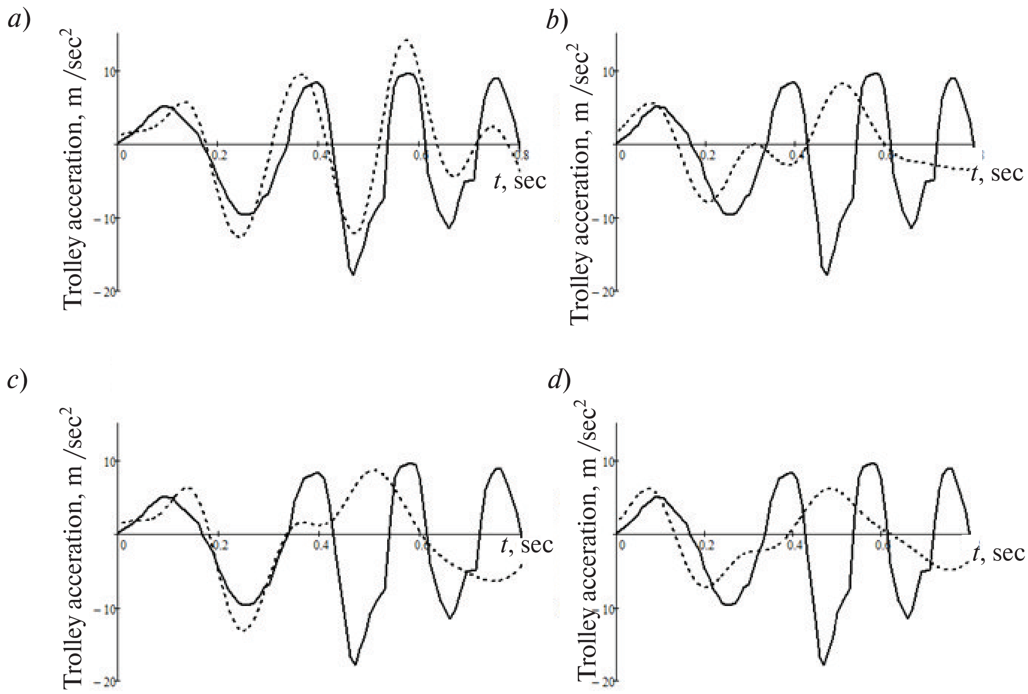

Fig. 5. Trolley accelerations versus time: ______ experiment; .... model(*a* – physicall and geometricfll nonlinear; *b* – geometricall nonlinear; *c* – physicall nonlinear; *d* – linear)

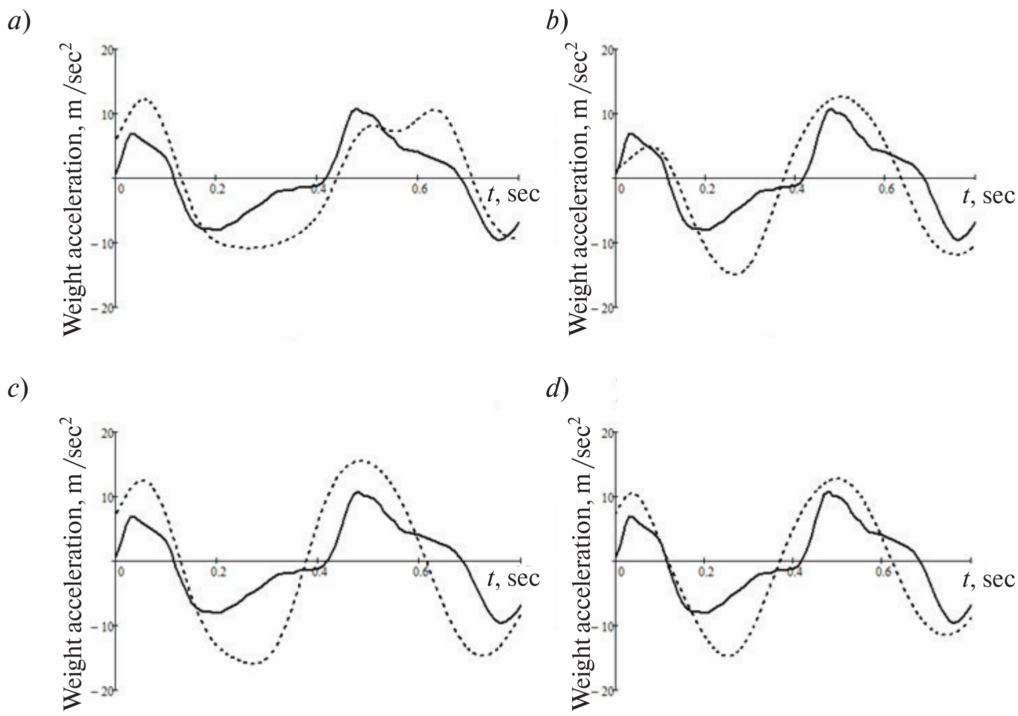

Fig. 6. Weight accelerations versus time: ______ experiment; .... model (*a* – physicall and geometricfll nonlinear; *b* – geometricall nonlinear; *c* – physicall nonlinear; *d* – linear)

Table

Parameter	Amplitude of bridge displacements		Amplitude of trolley accelerations		Amplitude of weight accelerations		Bounce duration	
	Value, mm	Deviation, %	Value, m/s^2	Deviation, %	Value, m/s^2	Deviation, %	Value, s	Deviation, %
Experiment	86,3	–	17,8	–	18,8	–	0,06	–
Nonlinear model	86,4	0,1	22,2	24,7	18,9	0,5	0,065	8,3
Linear geometry	96,3	11,6	21,8	22,5	31,4	67,0	0,146	143,3
Linear stiffness	76,1	-11,8	16,1	-9,6	27,5	46,3	0,038	-36,7
Linear model	76,5	-11,4	13,3	-25,3	27,5	46,3	0,023	-61,7

It is evident from the results of the study that the nonlinear mathematical model best describes the oscillations of the experimental setup. The linearization of the model leads to a significant deviation in the criteria for comparing the models and changes the behavior of the oscillatory process in the system. Without a doubt, the seismic calculation of a crane is not the most accurate type of analysis since it is impossible to predict the input effect. Therefore, this calculation should be carried out with a safety margin, with some results deliberately set too high. The choice of the mathematical model and its linearization are of interest. Let us consider the options for linearizing the mathematical model used for calculating the seismic stability of the cargo crane, based on the results of the experimental study.

Steel wire ropes used in hoisting equipment often have stiffnesses significantly higher than that of the crane structure. The safety factor adopted for the ropes is usually in the range of values $n = 4...25$ [9] depending on the operation mode of the crane, the purpose of the rope, and other characteristics of the particular installation. As shown in Ref. [10], the nonlinear properties of the ropes manifest themselves at relatively high loads with respect to the breaking force, if the safety factor is close to 3. In view of the above, modeling the nonlinear properties of steel wire ropes while calculating the seismic resistance of general-purpose cargo cranes in the absence of additional requirements for the calculation accuracy can be deemed ineffective.

Geometric linearization of the model in this study led to an overestimation of all evaluation criteria, and the behavior of the oscillations in the system changed to a large extent.

Constructing a physically and geometrically nonlinear model of a cargo crane is not economically feasible. A project designer does not require the bulk material of the supporting structures to be physically nonlinear, since the situation when the calculated stresses in the metal are located in the yield region is inadmissible. Modeling the nonlinear properties of steel wire ropes is also infeasible for the reasons described above. However, geometric linearization of the crane structure would unreasonably increase the safety factors that would in turn increase the overall costs of the crane.

Conclusions

The following conclusions can be reached from the results of the study conducted:

The experimental study has shown that physical and geometrical linearization of the mathematical model describing the process of free damped oscillations of a dynamical three-mass system causes the calculated displacements and accelerations to deviate from the true values by 11%-67%.

Physical linearization of the stiffness of hoist ropes is the most preferable from the standpoint of labor costs and feasibility, as it has no significant impact on the result of the dynamical analysis.

It can be concluded from the results of the study that unilateral constraints (steel wire ropes, the wheel-rail contact) should be taken into account when calculating the seismic resistance of general-purpose hoisting equipment.

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СВЕДЕНИЯ ОБ АВТОРАХ/AUTHORS

GAVRILOV Petr A. – Peter the Great St. Petersburg Polytechnic University.
29 Politechnicheskaya St., St. Petersburg, 195251, Russia.
E-mail: issahel@mail.ru

ГАВРИЛОВ Петр Александрович – аспирант Санкт-Петербургского политехнического университета Петра Великого.
195251, Россия, г. Санкт-Петербург, Политехническая ул., 29.
E-mail: issahel@mail.ru